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SHORTER COMMUNICATIONS

THERMAL CONDUCTIVITY CALCULATION OF POROUS MEDIA BY MEANS OF NOMOGRAPHS

ADAM ZANKER
Haifa, Kiriat-Jam "G", Israel

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ALL INSULATING structures are porous; they contain voids, commonly filled with air. In bricks, for example, the voids are either pockets or cells, more or less spherical in shape, not exceeding 5 mm dia. approximately. In other kinds of insulators, like glass wool, asbestos or sand—there are recesses between particles or granules of solids.

The conductivity of such structures will depend upon the conductivities of the solid and of the pores, and partially upon the size, shape and distribution of the pores.

To predict exactly the conductivity of an insulator from these factors is probably impossible, but a reasonable approximation is however possible, and may serve, not to replace experimental values, but to explain them, and to give a conductivity values close enough to those experimentally measured.

The Russel's equation, which tries to predict the conductivity of porous materials, is based on the following assumption: The pores are cubes, all of the same size, with solid walls of uniform thickness, and the isothermal surfaces are planes. By the laws of conductors, in series and in parallel, and simple mathematics, the conductivity of the (cellular) insulator is found to be given by the following formula [1]

$$K_{por} = K_{cont} \times \frac{\dfrac{K_{dis}}{K_{cont}} \times \phi^{2/3} + 1 - \phi^{2/3}}{\dfrac{K_{dis}}{K_{cont}}(\phi^{2/3} - \phi) + 1 - \phi^{2/3} + \phi}$$

where

K_{por} = thermal conductivity of porous medium, in W/m°K

K_{cont} = thermal conductivity of continuous phase, in W/m°K

K_{disp} = thermal conductivity of dispersed phase, in W/m°K

ϕ = volume fraction of dispersed phase ("porosity").

The other case is, when the granules are solid cubes, all of the same size, separated by layers of air of uniform thickness. The structure is thus the reverse of the first case. This assumption is not very sound, as the granules in any real structure cannot be separated, but must touch each other. They will, however, touch in a limited number of points. For this ideal structure, the conductivity is given by the same type of equation, the first equation being modified by using the porosity ϕ as a volume fraction of obstacles, instead of a volume fraction of voids, as in the first equation.

As it was mentioned before, the porous media, used as insulators, have in general two kinds of structures:

(a) Cellular structure (like bricks, pumice, sponge, etc.)
(b) Granular structure (like sand, asbestos, glass wool, etc.).

The porosity ϕ in Russel's equation means as follows:

(a) at cellular structures: the porosity is a volume fraction of voids (mostly filled with air)
(b) at granular structures: the porosity is a volume fraction of obstacles.

It is obvious, that when using the Russel's equation for cellular structure, the thermal conductivity K_{cont} means the conductivity of solid and the K_{disp} means the conductivity of air (or another gas, or liquid).

The situation is reversed, of course, when the calculated insulator has a granular structure. Thus, the continuous phase is air (or other gas, or liquid) and the dispersed phase is a solid. Therefore, the values of K_{cont} and K_{disp} have to be used accordingly.

The volume fraction of dispersed phase may be calculated from the following formulae:

(a) $\phi_1 = \dfrac{\rho_{sol} - \rho_{por}}{\rho_{sol} - \rho_{(g.L.)}}$ = volume fraction of voids, in cellular structure

(b) $\phi_2 = 1 - \dfrac{\rho_{sol} - \rho_{por}}{\rho_{sol} - \rho_{(g.L.)}}$ = volume fraction of obstacles in granular structure

where: ρ_{sol} = density of solid component in porous structure, in kg/m^3,

ρ_{por} = density of porous material ("bulk density") in kg/m^3,

$\rho_{(g.L.)}$ = density of gaseous or liquid component in porous structure, in kg/m^3 [2].

The suitability of Russel's equation may be best shown on the practical examples:

Example No. 1. *Porous brick* (Cellular type material). The thermal conductivity of air at 30°C = 0·0255 W/m°K. The conductivity of solid matter of brick at zero porosity, may be assumed as = 1·3 W/m°K. The estimated volume fraction of dispersed phase $\phi = 0{\cdot}8$.

Result. Calculated from Russel's equation = 0·215 W/m°K.
Measured = 0·21 W/m°K.

Example No. 2. *Wooden (pine) sawdust* (Granular type material). Thermal conductivity of air at 20°C = 0·023 W/m°K. Thermal conductivity of pine wood (average) ≈ 0·15 W/m°K. Volume fraction of dispersed phase ≈ 0·45.

Result. Calculated = 0·07 W/m°K
Measured = 0·06 W/m°K

Example No. 3. *Glass wool* (similar to granular type). Thermal conductivity of air = 0·0255 W/m°K. Thermal conductivity of glass ≈ 0·74 W/m°K. Density of glass ≈ 2500 kg/m^3. Density of glass wool ≈ 200 kg/m^3. Volume fraction of dispersed phase (calculated from densities) $\phi = 0{\cdot}08$.

Result. Calculated thermal conductivity = 0·038 W/m°K. At 30°C, measured thermal conductivity = 0·037 W/m°K.

As it may be observed from these examples, the Russel's equation works satisfactorily in a great number of cases.

It was generally found, that for cellular materials (with solid, as the continuous phase) average and maximum prediction errors appear to be ≈6 and ≈15 per cent [2].

LIMITATIONS

The Russel's equation is based on the thermal conductivity only. The heat transfer due to convection may be neglected in all cases, when the pore diameter is less than 5 mm, and it occurs in the majority of cases with porous insulators.

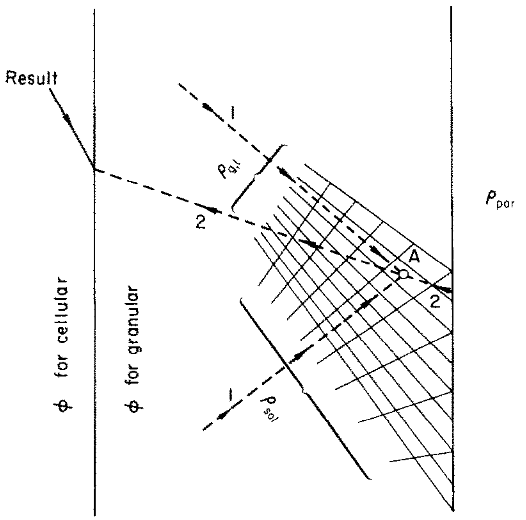


FIG. 1.

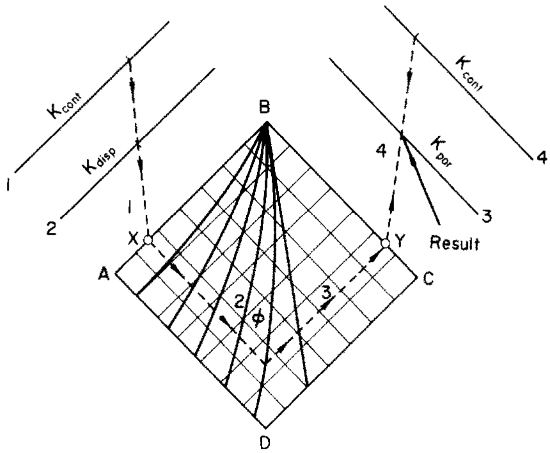


FIG. 2.

The heat transfer due to radiation is negligible as well, when the pores are small, and the temperature is low. However, as the radiation rises proportionally to the fourth power of temperature, its values become noticeable in temperatures, exceeding 400°C. For example, the heat transfer due to radiation in air pores reaches about 30 per cent of its conductivity value at 400°C; at 800°C, the radiation heat transfer is approximately equal to the heat transfer due to the conductivity at pores with about 0·3 mm dia. This phenomenon is even more noticeable when the pores are large; so that the conductivity of material with large pores increase with the temperature more rapidly, than for material with small pores [1].

Therefore the Russel's equation, which gives a very good approximation at relatively low temperatures, gives too low values at high temperature range.

The correction for heat transfer due to radiation is available, but not included into the attached nomographs, in order not to make them too complicated [3].

In some special cases, due to high permeability of porous material, the important role plays the vapour diffusion. For example, in the case of snow the diffusion mechanism causes a transfer of latent heat of sublimation. Therefore the thermal conductivity of snow may not be calculated by means of Russel's equation, without use of special correction factors [4].

Other observations showed that when the porous medium (granular type) is a very fine powder, the results obtained from the equation are too low.

Laubitz has shown, that doubling the right side of the equation restores its accuracy in this case [3].

In spite of the above mentioned limitations, the Russel's equation may be very useful in a great number of cases.

THE NOMOGRAPHS

The three attached nomographs for solving graphically the Russel's equation are as follows:

(1) *The nomograph for volume fraction of dispersed phase calculation*

This nomograph is used for the calculation of the volume fraction of dispersed phase ("porosity") of porous media (granular and cellular types) on the basis of known densities of both components (solid and gas or liquid), as well as the density of porous material ("bulk density").

(2a) *The nomograph for thermal conductivity calculation of porous media (Part I—Granular)*

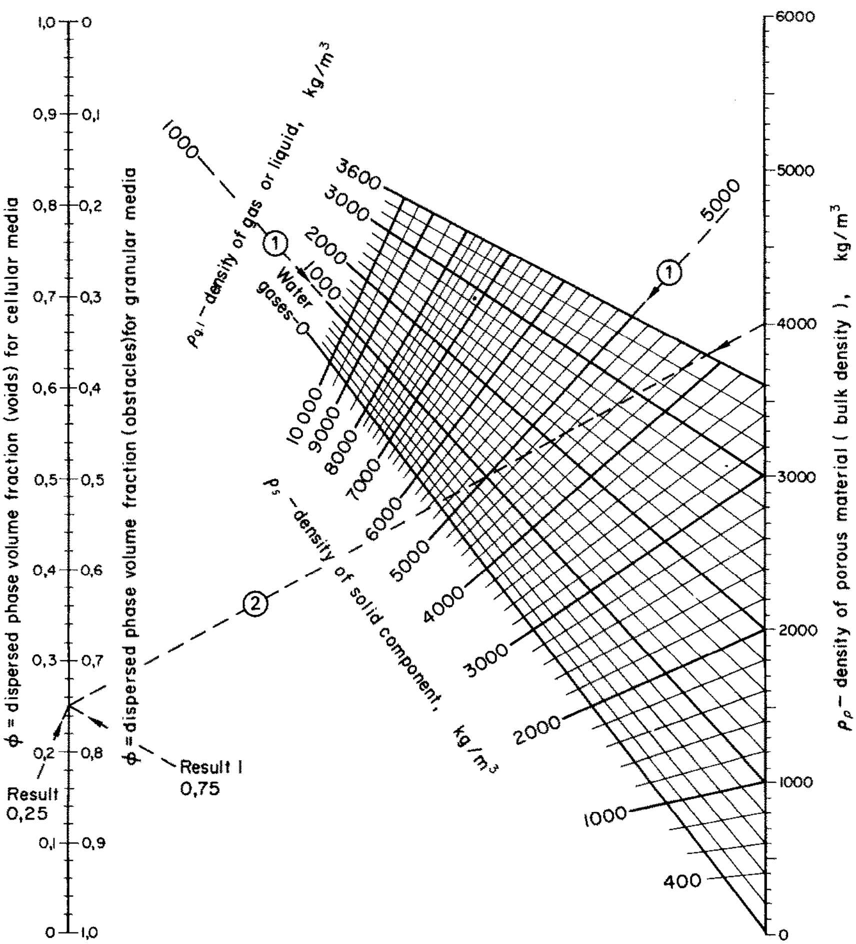
(2b) *The nomograph for thermal conductivity calculation of porous media (Part II—Cellular)*

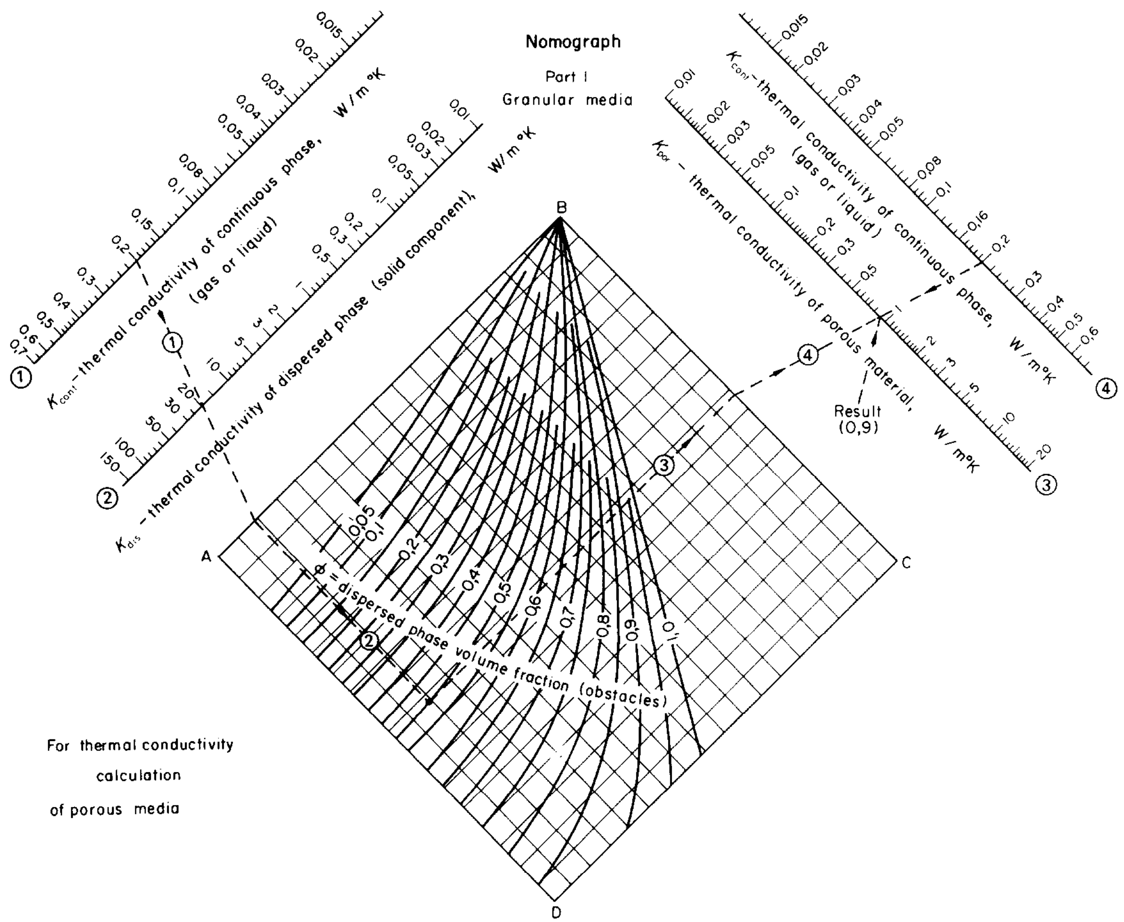
Both nomographs are based on the known values of thermal conductivities of continuous and dispersed phases, and on the value of volume fraction of dispersed phase, obtained previously by means of the nomograph No. 1, or estimated by another method.

Nomograph

For the calculation of dispersed phase volume fraction (porosity)

For granular and cellular porous media





HOW TO USE THE NOMOGRAPHS

The nomograph for calculation of volume fraction of dispersed phase

(1) Find the intersection point of two lines of known values of densities ρ_{sol} (for solids), and $\rho_{g.L.}$ (for gas or liquid) in the binary field; mark this point as A.

(2) Connect the point A with the known value of ρ_{por} (bulk density) on the right vertical scale, using a ruler. Extend this line up to the intersection point with the left, vertical ϕ scale.

The result (volume fraction of dispersed phase) has to be read on the ϕ scale: On the left-hand side of the scale, when the material is cellular, and on the right-hand side, when it is granular.

Typical examples

Granular type medium. What will be the porosity (i.e. volume fraction of obstacles) of a porous material, when the density of solid $\rho_{sol} = 5000$ kg/m^3, the density of liquid (continuous phase) $\rho_{g.L.} = 1000$ kg/m^3, and the density of porous material $\rho_{por} = 4000$ kg/m^3?

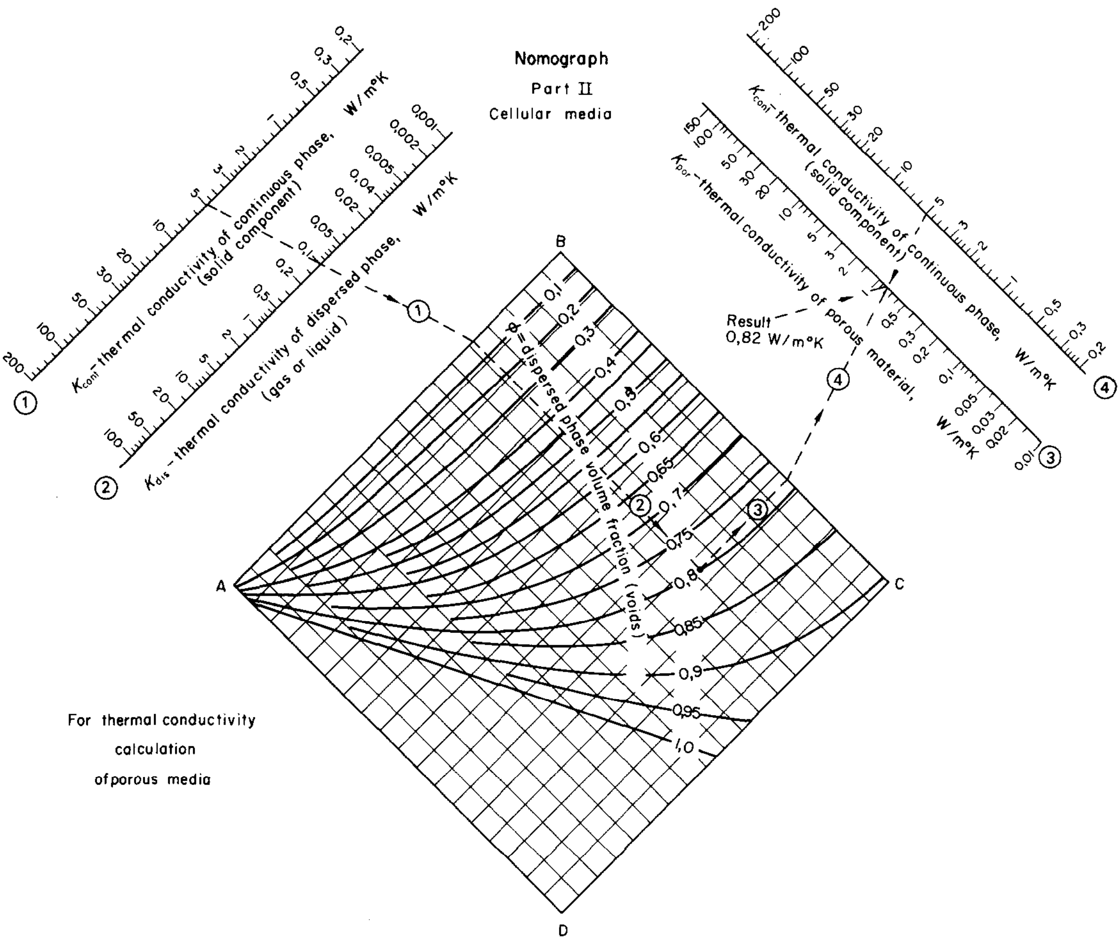
Result—The porosity is 0·75.

Cellular type medium. What will be the porosity (i.e. volume fraction of voids) of a porous material, when all the densities are as at the Example above, the solid being only continuous phase?

Result—The porosity is 0·25. (Both these examples are drawn with a dotted line on the attached nomograph).

The nomographs for thermal conductivity calculations (Part I—Granular) and (Part II—Cellular)

(1) The known values of thermal conductivities K_{cont} and K_{disp} on the scales (1) and (2) have to be connected with a ruler, and this line extended up to the intersection with the side AB of the square (point X).



(2) From the point X draw a line, parallel to the side BC of the square, until it meets the curve of given ϕ value.

(3) From this point draw a line, perpendicular to the previous one (i.e. parallel to the $\overline{AB}$ side of the square) until it intersects the $\overline{BC}$ side of the square (point Y).

(4) Connect point Y with a ruler, with the known value K_{cont} on the scale (4). The intersecting point of a ruler with the K_{por} scale [No. (3)] gives us the final result (i.e. the thermal conductivity of porous medium).

Typical examples

For granular media (*Nomograph marked Part—I*). What is the thermal conductivity of a porous (granular) material, when

$K_{cont} = 0{\cdot}2$ W/m °K, $K_{disp} = 20$ W/m °K and $\phi = 0{\cdot}5$.

Result—The thermal conductivity of porous (granular) material is $K_{por} = 0{\cdot}91$ W/m °K.

For cellular media (*Nomograph marked Part—II*). What is the thermal conductivity of porous (cellular) material when $K_{cont} = 5$ W/m °K, $K_{disp} = 0{\cdot}1$ W/m °K and $\phi = 0{\cdot}8$.

Result—The thermal conductivity of porous (cellular) material is $K_{por} = 0{\cdot}82$ W/m °K. (Both these examples are drawn with the dotted lines on both attached nomographs.)

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